WORKING PAPER

A Monocentric Analysis
of Housing Budget Restrictions,
Including and Without Transportation

Nicolas Coulombel[a]

[a] Université Paris-Est, Laboratoire Ville Mobilité Transport, 19 rue Alfred Nobel, Cité Descartes – Champs sur Marne, F-77455 Marne la Vallée Cedex 2, FRANCE

Contact Information for the Corresponding Author:

E-mail: nicolas.coulombel@enpc.fr

Tel: (+33) 1 64 15 21 30

Fax : (+33) 1 64 15 21 40

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Corresponding Author: Nicolas Coulombel

ABSTRACT

Considering the last surge in fuel prices, the policy to limit the share of housing expenses in the households' budget, so as to secure their solvency, has been criticized. Supposedly, it induces people to get farther from the city center in search of cheaper housing prices, but with subsequent increased transportation costs that are often disregarded during the house search process. To address this issue, several researchers have advocated to set a constraint on the share of both housing and transportation expenditure.

The present paper analyzes and compares the effects of the two policies on the main features of the city and on the households' utility. The analysis is carried out within the classical monocentric model of urban economics. After a general analysis, an applied model is specified to capture the effects of each policy in straightforward formulae.

I find that constraining housing expenses may increase the well-being of households. Additionally, both policies prove to be effective in reducing urban sprawl and thereby energy consumption. Thus the choice of the optimal policy will depend on the local authorities' objectives.

Keywords: monocentric model, urban economics, housing expenses, transportation expenses, housing policy, location efficient mortgage

INTRODUCTION

During the 2008 surge in oil prices, notable concerns rose about the "solvency" of households, which I define here as their ability to meet all their expenses[1]. This was especially the case in tight housing markets, where households have to face significant housing expenditures. And although the subsequent drop has somehow relieved the households' budgets, concerns remain over the long-term situation since oil prices are more than likely to be on the rise again. Under such circumstances, the relevance of capping housing expenditures at a given fraction of the household income, measure which had already been questioned, has become even more controversial[2]. Such practice is common in several countries in order to preserve the household solvency. In France it is enforced in two ways:

- Monthly payments for home loans amount to at most one third of the household income (28 % in the U.S. according to Duca and Rosenthal (1994)).
- When applying to rent a home, candidates must earn at least around three times the required rent[3].

While this policy does seem to secure the solvency of the households, it may spur them to settle far from the center of the agglomeration in search of moderate housing prices. Such is the case in the Greater Paris Region, whose central part desperately lacks affordable housing supply. This induces new homeowners to settle farther and farther in the suburbs, thereby contributing to urban sprawl (Polacchini and Orfeuil (1999)). Furthermore, because suburban households usually make the most extensive use of the car, we will see that they expose themselves to significant transport costs, which combined to the housing burden jeopardize the household budget. To prevent such undesirable collateral effects, several researchers (Hare (1995), Polacchini and Orfeuil (1999)) have advocated the equivalent of a joint budget constraint (housing plus transportation) for homebuyers, instead of the current practices. Their aim is twofold:

- To increase public awareness of the extent of transportation costs implied by suburban and exurban lifestyles.
- Making near transit locations more affordable by increasing the size of the home loan for households willing to locate in such areas (based on future savings on transportation).

This idea was implemented in the U.S., under the name of "Location Efficient Mortgage"[4], but only in a limited number of housing markets.

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[1] This definition therefore covers the usual notion of solvency as the ability of households to meet their financial obligations on time, and in particular mortgage-related ones.

[2] For the reminder of the text, we will equally use the terms "burden" or "expense ratio" to refer to the fraction of the household income dedicated to a budget item, e.g. housing or transportation. The housing expense ratio is also sometimes referred to as the front ratio.

[3] The ratio of one to three corresponds to a widespread practice in the Paris Metropolitan Area, though few renters may even require up to four times the rent. In the reminder of France, income requirements may be less strict.

[4] See www.locationefficiency.com for more on the LEM, which notably stemmed from the work achieved by Haas et alii (2006)

Although there is abundant economic literature assessing land-use regulatory policies (e.g. Bertaud and Brueckner (2005) or Brueckner (2006)), this is not the case for the specific policies I have mentioned. I propose to remedy this gap by analyzing first the policy limiting the housing expense ratio (which I call the Constrained Housing Expense (CHE) policy), then the one capping the total share of transportation and housing expenses (the Constrained Housing+Transportation expenses (CH+T) policy). The analysis is carried out within the classical framework of urban economics, the monocentric model. The impacts of each policy on the main features of the city are brought to light, then compared; in particular I examine the issue of the well-being of the households, the city size and the related transport costs, and rent prices. As will be seen, both policies reduce urban sprawl (and thus could contribute to reduced energy consumption) while maintaining or even increasing the well-being of the households.

I shall present this work as follows: the first section being the present introduction, section two describes the context and the scope of the study. Sections three and four respectively analyze at great length the CHE and CH+T policies. Section Five offers by way of conclusion a comparative analysis of the two measures, and policy recommendations.

CONTEXT AND SCOPE OF THE STUDY

As I mentioned in the introduction, no economic work has specifically tackled the issue of assessing the CHE and CH+T policies. Still, three sets of works bring useful insights to the topic of this study. Along the presentation of these works, I first provide indirect yet conclusive evidence for the significant influence of the CHE policy. It will also be shown that CHE and CH+T policies more specifically target low- and middle-income households. Next, a survey of existing works on CHE and CH+T policies is carried out. Finally, I present the framework of analysis, and specify the scope of the study.

On housing and transportation burdens

A first set of empirical works allows grasping the size of the issue at stake by focusing on the households' housing and transportation burdens. As a matter of fact, three questions are preliminary to the present study:

1. Does the CHE policy concern a significant number of households?
2. Is the impact on housing choices noteworthy?
3. Do the spatial variations of transport costs really matter in front of the housing burden?

By providing estimates of the housing and transportation burdens, Polacchini and Orfeuil (1999), Berri (2007) and Coulombel, Deschamps and Leurent (2007) bring first pieces of answer to question 1 and 3 for the Paris Metropolitan Area (PMA). Despite differences in methodology or in the year of interest, all works draw similar conclusions regarding the housing and transportation expense ratios:

- The housing expense ratio is fairly stable over space, and close to the maximum allowed by the CHE policy. Polacchini and Orfeuil (1999) find for the year 1991 an average front ratio of 32% for homebuyers and 26% for tenants of the private market. Coulombel, Deschamps and Leurent (2007) respectively find 28% and 39% for year 2001. Berri (2007) provides the lowest

housing burdens, with 28% for homebuyers and 22.4% for tenants of the unregulated sector in 1994.

- On the other hand, the transportation burden sharply increases with distance to the Central Business District (CBD), as a result of a higher car modal share in the suburbs, as well as households making longer trips. Coulombel, Deschamps and Leurent (2007) have found that expense ratios range from 7% for inner Paris to 21% for the most remote parts of the PMA.

Given these two facts, all works bring to light an increasing trend for the overall housing and transportation burden.

Interestingly Haas et alii (2006) reach similar conclusions for the U.S. despite notorious differences with Europe regarding urban structure: analyzing 28 metropolitan areas, they find the housing burden to be much less sensitive to location than the transportation burden, which strongly increases with distance to the nearest employment center. For households with yearly income between 35,000 and 50,000$, the average housing burden varies between 23 and 26% according to the location within the metropolitan area, against 16 to 26% for transportation.

These findings naturally lead to the two following statements[5]:

- The relative constancy of the housing expense ratio over space (within a given metropolitan area), combined to its closeness to the theoretical upper bound, is most likely the effect of the CHE policy.

- Given this constancy, the increasing trend of the transportation share jeopardizes suburban households, who face a heavy joint housing & transportation burden (sometimes exceeding half their income).

Two elements support the first statement, which might not come out as obvious at first. Firstly, housing burdens display volatility: thus an average housing burden not far from the theoretical upper bound most probably comes with a sizable number of constrained households. Secondly, exhibiting a front ratio lower than the upper theoretical bound does not automatically imply that the household was not constrained by the CHE policy when it made its housing choice (i.e. when it acquired or rented its present dwelling). Indeed, income often rises during the household life-cycle, notably because of inflation and job promotions. Besides, when the household has successfully reimbursed its mortgage, its housing burden also drops. Consequently one household usually sees its housing burden progressively decline until its next residential move[6]. This could partly account for the fact that several households display housing expense ratios lower than the upper bound.

Although the two previous assertions would tend to motivate the present study by providing indirect evidence for the significance of the CHE policy, one has to moderate them by underlining the role of income in the previous analyses. As a matter of fact, all works accounting for income (Berri (2007), Coulombel, Deschamps and Leurent (2007) and Haas et alii (2006)) put forward the significant decrease of both the housing and the transportation burdens with

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[5] While these statements are not explicit in Haas et alii (2006), such is the case in Polacchini and Orfeuil (1999), Berri (2007) and Coulombel, Deschamps and Leurent (2007).

[6] In fact, the housing burden may also increase in case of an adverse event on the job market such as unemployment, or if the housing expenses increase due to specific conditions (renegotiation of the lease, flexible interest rate mortgage products...).

income[7]. Low- and middle-income households are thus more likely to face a heavy H+T burden than high-income households. This also implies that high-income households are less likely to be constrained by CHE or CH+T policies.

To conclude on this first statement, let us put forward additional figures which prove to be enlightening. In a study on the effects of borrowing constraints, Gobillon and Le Blanc (2008) estimate thanks to an econometric model that 53% of the tenants of the private sector would be constrained were they to opt for ownership[8]. Although the share of potentially constrained households is logically lower for homeowners (homebuyers and outright owners confounded), it still adds up to 20% of this category. If one adds up all the previous elements, the significance of the CHE policy in the case of homebuyers is clearly established. As regards tenancy, although previous studies have emphasized significant housing burdens, which are likely to come along with a substantial fraction of constrained households, a more precise assessment of the phenomenon has yet to be made.

Similarly, the second statement may seem peculiar at first. A sound economic reasoning would raise the fact that rational households with rational expectations freely and adequately choose their housing and transportation bundle. This implies that the high housing plus transportation share of suburban households is a choice and not a danger, even if it were to represent more than half the household income. Yet three arguments challenge this line of thinking:

- The housing market might not be perfectly competitive. In case of sticky prices[9], "insider households" in the center of the agglomeration might want to stay in order to enjoy low transportation costs, leading "outsider households" to be restricted to suburban locations with housing prices not compensating the heavy transportation burden. In such a case, stickiness of prices would prevent the adjustment of housing prices in the central part of the agglomeration given the strong demand for these locations.
- Households might not be perfectly informed of transportation costs. This might especially be the case for car-owners: the presence of fixed and variable costs, the issue of maintenance, the cost of the credit (when applicable), the possibility of selling the car to get a new one, are all elements that contribute to a difficult perception of the true cost of a car[10]. The variability of fuel prices might also not be well apprehended. Besides, many households do not consider fixed costs in the equation because they take the fact that they need a car for granted, and thus compare the cost of transit to the variable cost of private transportation.

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[7] Let us note that accounting for the household income does not alter the spatial patterns of the housing and transportation expense ratios.

[8] To be more precise, by predicting the value of the dwelling the household would be willing to purchase if it were to opt for ownership at a given time t, the econometric model developed by the authors can estimate the number of households who would face borrowing constraints given their current income and wealth. The two main types of borrowing constraints are considered, i.e. the income constraint we mentioned and the upfront payment constraint. Gobillon and Le Blanc (2008) point out that the income constraint prevails in most cases, corroborating the significance of the CH policy.

[9] The assumption of sticky prices in the housing market should be validated but the regulation of rents operating in several countries strongly supports this idea.

[10] From my personal experience, many people around me have no idea of the extent of fixed costs involved by the ownership of a car on a yearly or on a per km basis.

- A last argument relying on moral hazard is that households might not sufficiently consider the issue of bankruptcy (from a point of view optimizing social welfare) because of laws and public policies protecting financially distressed households.

Overview of the existing literature on CHE and CH+T policies

While there are few works on the regulation of housing expenses in the rental market[11], the effects of borrowing constraints on the demand for housing have largely been documented by the economic literature. These works, described at great length in the survey carried out by Gobillon (2008), focus on the household decision to move and on the subsequent choice of tenure. Typically, households are assumed to opt for both tenure and the quantity of housing stock (or housing service in the case of tenancy) so as to maximize their utility. Moving costs and transaction costs are introduced in most models to account for the fact that housing adjustments (which occur under the form of a residential move[12]) do not happen constantly, but only when housing consumption gets too far from its optimal value. When a move occurs, the household chooses simultaneously its type of tenure according to the relative current and future prices of renting and owning, and its ideal quantity of housing consumption. Because borrowing constraints prevent concerned households from choosing their optimal value of housing stock, it has the double impact of making tenancy more attractive and hindering residential mobility. The latter effect might even prevail according to Zorn (1989) or Gobillon and Le Blanc (2008).

This literature has shed important light on the household behavior in front of borrowing constraints. It has also collected enough evidence to answer positively to above question 2.[13] Yet, it fails at answering our questions for two reasons. First, most works do not consider the housing supply side, and consequently equilibrium mechanisms. The impacts of borrowing constraints on housing prices are usually beyond scope. The omission of space is another significant weakness of most works on this topic, as well as of the few works that specifically deal with the issue of location efficient mortgages[14]. Since housing prices vary within the metropolitan area, borrowing constraints are likely to deeply influence the location choices of households. According to Hare (1995), what he calls "clunker mortgages" are even central in accounting for urban sprawl.

Theoretical framework

Given the previous remarks, a better understanding of the effects of CHE and CH+T policies implies considering the role of space and equilibrium mechanisms. This is where a third part of the economic literature, which focuses on the analysis of land-use regulatory policies, proves useful. Based on the use of the classical framework of urban economics, namely the urban monocentric model[15], several works have studied the effects of restrictions on city size, density (or alternatively building-height with the introduction of maximal or minimal Floor Area Ratios),

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[11] It is important to note that regulating housing expenses is different from rent control: the CHE policy operates at the household level and not at the dwelling level with rental price ceilings like it is the case for rent control.

[12] The possibility of maintenance (or more generally house works) as a form of stock adjustment is rarely considered in this literature.

[13] Once again we refer the reader to Gobillon (2008) for conclusive evidence on this issue.

[14] E.g. Blackman and Krupnick (2001)

[15] See Fujita (1989) for a very thorough analysis of this model.

as well as other forms of regulation. The ability of the monocentric model to represent both the demand and supply side of the housing market within a spatial framework makes it an adequate tool for the analysis of such policies. Recent contributions of Bertaud and Brueckner (2005) and Brueckner (2006) afford a good overview of this significant body of the urban economic literature.

Because the monocentric model has been shown to be particularly suitable to study housing or land use policies in a spatial equilibrium setting, I have chosen it for the present analysis and I will now outline its main characteristics. In the version of the model that I am going to use, households with income Y maximize their utility $U(z,s)$ through a tradeoff between two goods, land (s representing land consumption or lot size) and a composite good denoted by z standing for all other goods, under a budget constraint. This economic behavior is represented by the following maximization problem:

$$\underset{z,s,r}{Max}\ U\ (z\,,\,s)\ \ s.t.\ \ \ R\,(\,r\,)\,s\,+\,z\,+\,T\,(\,r\,)\,=\,Y$$

While $R(r)$ stands for the relative land rent, z is the numéraire good, and $T(r)$ represents transport costs. The variable r represents location: since locating farther from the central business district (CBD) implies higher transport costs, households typically trade-off between accessibility and housing prices when choosing their location. The essence of this model lies in the endogeneity of housing prices, which vary according to the law of supply and demand. At equilibrium, prices reflect the "spatial advantage" of a given location.

Scope of the study

The choice of this specific version of the monocentric model holds several assumptions, which I am now going to discuss. This will also give us the opportunity to specify the scope of the present study.

Transportation network

Several assumptions are made about the transportation system:

(H1)   The transportation network is assumed to be "unimodal" and dense.

(H2)   Transportation costs only include monetary costs.

(H3)   They are isotropic and determined only by location.

(H4)   Transportation costs increase with distance.

Among the four assumptions (H2) is the most natural for two reasons: firstly, only monetary costs are considered in the CHE and CH+T constraints. Secondly, even if travel-time costs were to be included, Coulombel, Deschamps and Leurent (2007) have established for the Paris Metropolitan Area that neither location nor household income have a significant impact on travel time budgets. (H4) is a usual assumption in a monocentric framework[16]; it was verified for the PMA by Coulombel, Deschamps and Leurent (2007).

Now let us turn to (H1) and (H3). While transportation costs slightly increase with income, this feature is neglected for the sake of simplicity. Besides this point, the strongest assumption is to my view that of "unimodality". It is important to note that within the stylized

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[16] This would not be the case in a polycentric framework.

model that I use, the so-called "unimodality" hypothesis does not necessarily imply one single mode throughout the whole city. It rather corresponds to the fact that one location equals one given amount of transportation costs. Transportation costs may correspond to transit costs in the central part and car costs in the suburbs without affecting the validity of the model. Nevertheless, households may not choose between different modes at a given location. Thus, the "unimodality" assumption could be reformulated as the fact that practices of mobility are completely determined by location. This is not far from being true, especially in the PMA: walk and transit prevail in the dense and usually congested areas, while the car often represents the only sensible option for households living in the suburbs. This assumption is also corroborated by recent findings from Haas et alii (2006): they establish that transportation costs are driven more by neighborhood characteristics than by household or income.

{   **PARTIE EN COURS**

Representation of the housing market

Pas de representation des offreurs (Muth) -> terrain=service lgt. Pas de varieties des lgts

Representation of the housing market: Note that in the simplified context of the monocentric model, land rents and housing prices are equivalent.

Representation of space

Ville linéaire et isotrope

Representation of the households

Unicité des preferences, du revenu, pas de prise en compte des caractéristiques du ménage

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CAPPING THE HOUSING EXPENSE RATIO

This section analyzes the impact of the CHE policy in terms of:

- Household utility
- Land use: city size, density
- Composition of the household budget

To do so, I first present the constrained housing expenses (CHE) model and solve the household maximization problem. Then I characterize the equilibrium city and proceed to comparative statics in the general case. Lastly, I study the different impacts of the CHE policy in the case of a linear city.

The Constrained Housing Expense (CHE) model

Let us consider the general case, where $U(z,s)$ and $T(r)$ are assumed to comply with only the classical hypotheses :

- The utility function $U(z,s)$ is concave, strictly increasing with z and s, and is well-behaved[17].
- Transportation costs $T(r)$ increase with distance r to the CBD.

Presentation of the CHE model

The CHE policy consists in capping housing expenditures at a given fraction of the household income. To study the effects of this policy I therefore amend the monocentric model with the following constraint:

$$R(r)s \leq \alpha Y \qquad (E1)$$

where $\alpha \in [0,1]$. Given the budget constraint of the household, $(E1)$ is equivalent to the following constraint, which will prove easier to handle:

$$z \geq (1-\alpha)Y - T(r) \qquad (E2)$$

Consequently, the household maximization problem becomes:

$$\max_{z,s,r} U(z,s) \quad s.t. \quad \begin{cases} z + R(r)s + T(r) = Y \\ z \geq (1-\alpha)Y - T(r) \end{cases} \qquad (E3)$$

Note that $\alpha=1$ yields the original unconstrained model.

Notation

The following notations are used throughout this section:

- A tilde superscript (~) for the CHE model, no symbol for the original model
- I often omit the argument α when unnecessary.
- $E_A(u,\alpha) = \{r \,/\, z(r,u) < (1-\alpha)Y - T(r)\}$ is the strictly binding zone, defined as the set of locations r where the Lagrange multiplier associated to $(E2)$ is strictly positive.
- $E_I(u,\alpha) = \overline{E}_A(u,\alpha)$ is the nonbinding zone[18], and $\breve{E}_I(u,\alpha)$ its open subset.
- $S(z,u)$ is the inverse function of $U(z,s)$ with respect to s.
- $Z(s,u)$ is the inverse function of $U(z,s)$ with respect to z.
- r_{max} is the farthest feasible location: $T(r_{max}) = Y$.

The bid-max program

Bid rent function of the household   Bid rent functions for the CHE and original models are defined as usual:

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[17] See definition provided in Fujita (1989) p.99

[18] Thus the complementary of $E_A(u)$, which is also the zone where the constraint is Inactive

$$\widetilde{\Psi}(r,u) = \max_{z,s}\left\{\frac{Y-T(r)-z}{s}\;\middle|\;\begin{matrix}U(z,s)=u\\z\geq(1-\alpha)Y-T(r)\end{matrix}\right\}$$

$$\Psi(r,u) = \max_{z,s}\left\{\frac{Y-T(r)-z}{s}\;\middle|\;U(z,s)=u\right\}$$

(E4)

Argmax of the unconstrained program are denoted $s(r,u)$ and $z(r,u)$. Let us recall the classical properties:

- $s(r,u)$ increases with r and u
- $\Psi(r,u)$ decreases with r and u
- $z(r,u)$ decreases with r

Because $\widetilde{\Psi}(r,u)$ is obtained by adding the HE constraint to the original program, we have the following property (proof omitted):

PROPERTY 1

$$\begin{cases}\widetilde{z}(r,u) = \max\left[z(r,u),(1-\alpha)Y-T(r)\right]\\\widetilde{s}(r,u) = \min\left[s(r,u),S\left((1-\alpha)Y-T(r),u\right)\right]\\\widetilde{\Psi}(r,u) = \min\left[\Psi(r,u),\alpha Y/\widetilde{s}(r,u)\right]\end{cases}$$

(E5)

which implies that $\forall(r,u)$, $\widetilde{z}(r,u) \geq z(r,u)$, $\widetilde{s}(r,u) \leq s(r,u)$ and $\widetilde{\Psi}(r,u) \leq \Psi(r,u)$

To sum up, for a given utility level, and inside the binding zone, capping housing expenditures reduces:

- the lot size which is bid for.
- the ability to pay for a unit of land.

Properties of the bid-max variables   A binding HE constraint alters the solutions. Nevertheless, system (E5) ensures that:

- $\widetilde{s}(r,u)$ increases with r, u and α
- $\widetilde{\Psi}(r,u)$ decreases with r, u and increases with α
- $\widetilde{z}(r,u)$ decreases with r and α

Conservation of the properties with respect to r and u is central to demonstrating the existence and uniqueness of the equilibrium land use. Regarding the role of α, relieving the constraint increases the maximum level of housing expenditures, which allows households to purchase bigger lots, increase their bid rent, and reduce their consumption of the z good.

The case of single household type

I investigate in this subsection the standard framework of a closed city with absentee landlords and inhabited by households of a given single type, with income Y and utility function $U(z,s)$.

After demonstrating the existence and uniqueness of the equilibrium in the CHE model, I perform comparative statics in order to compare the CHE equilibrium to the original equilibrium.

As usual, we note as N the number of households and we assume positive land supply $L(r)>0$ at all $r>0$.

Existence and uniqueness of the CHE equilibrium

As in Fujita (*9*) for the unconstrained model, demonstrating the existence and uniqueness of the equilibrium in the CHE model is equivalent to proving that there exists a unique couple $(\tilde{u}, \tilde{r}_f)$ that complies with the following system:

$$\begin{cases} \tilde{\Psi}(\tilde{r}_f, \tilde{u}) = R_A \\ \int_0^{\tilde{r}_f} \dfrac{L(r)}{\tilde{s}(r, \tilde{u})} \, dr = N \end{cases} \qquad (E6)$$

The first equality is the boundary condition that determines the edge $\tilde{r}_f$ of the city: at $\tilde{r}_f$ bid rent equates the opportunity cost of land, R_A. The second equality corresponds to the population constraint: integration of the density function within the city gives N, the total number of households. Note that density $n(r)$ is given by the available land supply divided by the land consumption per household, i.e. $n(r) = L(r) \div \tilde{s}(r, \tilde{u})$

PROPOSITION 1

The CHE monocentric model with single household type admits a unique equilibrium.

PROOF OF PROPOSITION 1

Similarly to Fujita (*9*), we consider the outer boundary function $\tilde{b}(u)$ characterized by $\int_0^{\tilde{b}(u)} \dfrac{L(r)}{\tilde{s}(r, u)} \, dr = N$. $\tilde{b}(u)$ determines the city size for a given target utility u. Since $\tilde{s}(r, u)$ exhibits the same required features as $s(r, u)$, that is to say $\tilde{s}(r, u)$ is decreasing in u, tends toward $+\infty$ when $u \to +\infty$ and tends toward 0 when $u \to -\infty$, we could proceed similarly to Fujita and show that $\tilde{b}(u)$ is well-defined on an interval]-∞,a[, where possibly $a=+\infty$. Besides, $\tilde{b}(u)$ strictly increases with u and ranges from 0 to $+\infty$ when u ranges from -∞ to a.

Then we consider $\tilde{R}_{Bound}(x) = \tilde{\Psi}(x, \tilde{U}(x))$ where $\tilde{U}(x) = \tilde{b}^{-1}(x)$ for $x \in [0, r_{max}[$. $\tilde{R}_{Bound}(x)$ is the land rent at the edge x of a city, the utility of which has been chosen so as to procure the required size x. Since $\tilde{b}(u)$ increases strictly with u, $\tilde{U}(x)$ also increases strictly with x, implying that $\tilde{R}_{Bound}(x)$ is strictly decreasing in x (remember that $\tilde{\Psi}(r, u)$ is decreasing in

both r and u). Since $\tilde{R}_{Bound}(r_{\max}) = 0$ and $\tilde{R}_{Bound}(x) \underset{x \to 0}{\to} +\infty$, the equation $\tilde{R}_{Bound}(x) = R_A$

admits one and only one solution $\tilde{r}_f$. Eventually, by taking $\tilde{u} = \tilde{U}(\tilde{r}_f)$, it is trivial to check that $(\tilde{u}, \tilde{r}_f)$ satisfies system (*E6*).

Comparative statics in the general case

I determine here the influence of the constraint parameter α on the equilibrium city.

City Size   Quite intuitively, the CHE policy reduces the city size:

PROPOSITION 2

For any set (N,Y,R_A) the size $\tilde{r}_f(\alpha)$ of the CHE city increases with α

PROOF OF PROPOSITION 2

Let us first show that the constrained boundary rent curve $\tilde{R}_{Bound}(x, \alpha_1)$ is below the second one, i.e.: $\tilde{R}_{Bound}(x, \alpha_1) \leq \tilde{R}_{Bound}(x, \alpha_2)$

As $\forall (r, u)$, $\tilde{s}(r, u, \alpha_1) \leq \tilde{s}(r, u, \alpha_2)$ then $\int_0^x \dfrac{L(r)}{\tilde{s}(r, u, \alpha_1)} dr \geq \int_0^x \dfrac{L(r)}{\tilde{s}(r, u, \alpha_2)} dr$.

Since $\int_0^{\tilde{b}(u,\alpha_1)} \dfrac{L(r)}{\tilde{s}(r, u, \alpha_1)} dr = \int_0^{\tilde{b}(u,\alpha_2)} \dfrac{L(r)}{\tilde{s}(r, u, \alpha_2)} dr = N$, this implies $\tilde{b}(u, \alpha_1) \leq \tilde{b}(u, \alpha_2)$,

which in turn implies that the inverse functions are in reversed order, that is to say $\tilde{U}(x, \alpha_1) \geq \tilde{U}(x, \alpha_2)$.

Using the inequality $\forall (r, u)$, $\tilde{\Psi}(r, u, \alpha_1) \leq \tilde{\Psi}(r, u, \alpha_2)$, we have:

$\tilde{\Psi}(x, \tilde{U}(x, \alpha_1), \alpha_1) \leq \tilde{\Psi}(x, \tilde{U}(x, \alpha_2), \alpha_1) \leq \tilde{\Psi}(x, \tilde{U}(x, \alpha_2), \alpha_2)$

$\Rightarrow \tilde{R}_{Bound}(x, \alpha_1) \leq \tilde{R}_{Bound}(x, \alpha_2)$

which is the claimed property. Considering this, demonstration of proposition 2 is straightforward since $\tilde{R}_{Bound}(\tilde{r}_f(\alpha_1), \alpha_1) = \tilde{R}_{Bound}(\tilde{r}_f(\alpha_2), \alpha_2) = R_A$.

Because $\alpha \geq 1$ yields the original model, proposition 2 unveils that the CHE city is smaller than the original one.

Equilibrium utility   While the analysis of equilibrium utility is more complex, the following proposition gives an insight:

PROPOSITION 3

For any couple $\alpha_1 < \alpha_2$, if the household located at the edge of the α_2 city spends less than $\alpha_1 Y$ on housing (i.e. $\tilde{r}_f(\alpha_2) \notin E_A(\tilde{u}(\alpha_2), \alpha_1)$), then the equilibrium utility $\tilde{u}(\alpha_1)$ of the α_1 city is superior to the equilibrium utility $\tilde{u}(\alpha_2)$ of the α_2 city.

PROOF

For a household located at $\tilde{r}_f(\alpha_2)$, we have the following relations:

- $\tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_2), \alpha_1) = \tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_2), \alpha_2)$ from $\tilde{r}_f(\alpha_2) \in \overline{E}_A(\tilde{u}(\alpha_2), \alpha_1)$

- $\tilde{\Psi}(\tilde{r}_f(\alpha_1), \tilde{u}(\alpha_1), \alpha_1) = R_A = \tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_2), \alpha_2)$ (boundary conditions)

- $\tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_1), \alpha_1) \le \tilde{\Psi}(\tilde{r}_f(\alpha_1), \tilde{u}(\alpha_1), \alpha_1)$ due to $\tilde{r}_f(\alpha_1) \le \tilde{r}_f(\alpha_2)$ (proposition 2)

By combining these relations, we have $\tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_1), \alpha_1) \le \tilde{\Psi}(\tilde{r}_f(\alpha_2), \tilde{u}(\alpha_2), \alpha_1)$, which implies $\tilde{u}(\alpha_1) \ge \tilde{u}(\alpha_2)$.

Proposition 3 shows specific conditions under which the equilibrium utility of the CHE city decreases with α. By choosing $\alpha_2=1$, it gives a simple condition, sufficient but not necessary, for the CHE city to display a higher equilibrium utility than the unconstrained city (with equilibrium utility u_{eq}). On the other hand, we will see in the application to come that the case $\tilde{u}(\alpha) < u_{eq}$ is possible when the constraint puts an excessive burden on the households.

Actually, the HE constraint induces two effects that alter the equilibrium utility level:

- Being constrained in their choices, households achieve a lower utility at a given location and land rent price
- But capping housing expenses has a depressing effect on bid prices, hence on land rents, which tends to increase the utility of the households

Depending on the relative magnitude of these two effects, the resulting utility level of the HE city is higher or lower than that of the original city.

Housing expenses   Determining the influence of α on housing expenses proves not trivial, because tightening the HE constraint may result in a lower utility level, which in turn may increase the housing expenses of unconstrained households. Nonetheless, when the equilibrium utility rises, it is possible to show that tightening the constraint always diminishes the total land rent distributed to the landlords. This also holds when the constraint is binding for the whole city.

Application to a linear city

Considering the limitations of the general case analysis, I now provide a special case as an illustration, with a log-linear utility function $U(z,s) = 1/2 \log z + 1/2 \log s$, linear transport costs $T(r) = ar$ and a linear city: $L(r)=1$. I did not choose a disk-shaped city (i.e. $L(r)=2\pi r$) since calculations prove more complex, especially for deriving analytical results.

Derivation of the equilibrium city

After determining the binding zone, I derive the different variables of interest, that is to say bid-max variables, utility level and city size, which I use in the next subsection to analyze the equilibrium outcomes.

Determination of the binding zone   The log-linear form utility function implying that $z(r,u) = 1/2(Y - ar)$, the housing expenditure constraint is strictly binding when:

$$r < r_{bind}(\alpha) = \frac{(1 - 2\alpha)Y}{a} \tag{E7}$$

Thus:

- if $\alpha \geq 1/2$ the HE constraint is never binding, the CHE model is equivalent to the unconstrained model.
- if $\alpha < 1/2$, only households located closer than $r_{bind}(\alpha)$ are effectively submitted to the HE constraint.

Characterization of the equilibrium   Resolution of the bid-max program brings about the following formulae:

$$r \leq r_{bind}(\alpha) \begin{cases} \tilde{z}(r,u) = (1 - \alpha)Y - ar \\ \tilde{s}(r,u) = e^{2u}/\tilde{z}(r,u) \\ \tilde{\Psi}(r,u) = e^{-2u}\alpha Y\{(1 - \alpha)Y - ar\} \end{cases} \qquad r \geq r_{bind}(\alpha) \begin{cases} \tilde{z}(r,u) = (Y - ar)/2 \\ \tilde{s}(r,u) = e^{2u}/\tilde{z}(r,u) \\ \tilde{\Psi}(r,u) = e^{-2u}(Y - ar)^2/4 \end{cases} \tag{E8}$$

Figure 1 illustrates these solutions for the following settings (which will constitute the reference model): N=10, Y=80, a=8 and R_A=20. In addition to that I choose α=0.20 and u=21.21 (which corresponds to the equilibrium utility of the CHE model for the chosen settings). For these settings r_{max}=10 and r_{bind}=6.
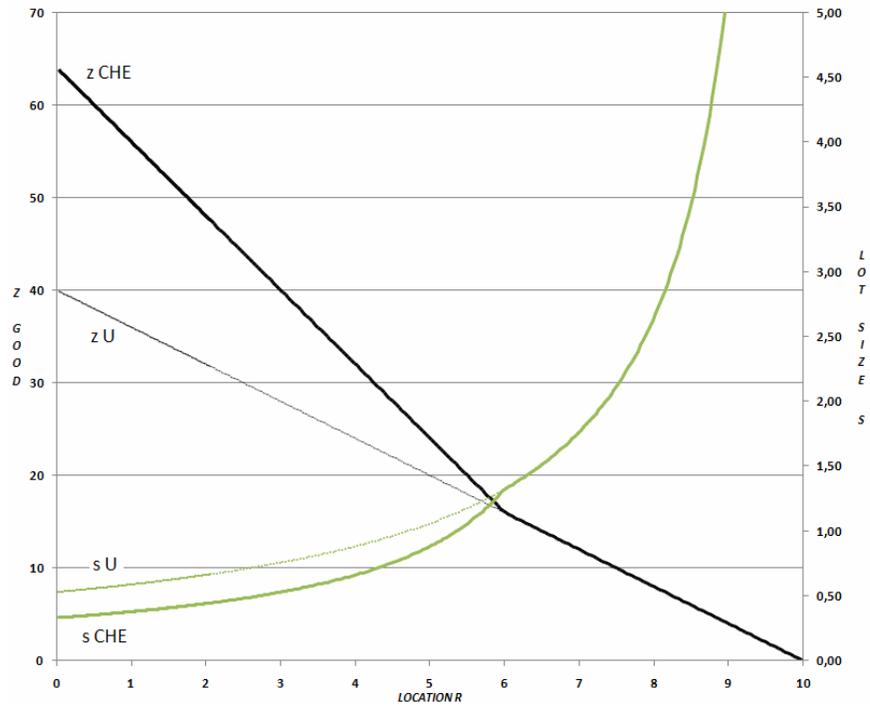
FIGURE 1a   Lot size and z good consumption in the Unconstrained (U) and CHE models.
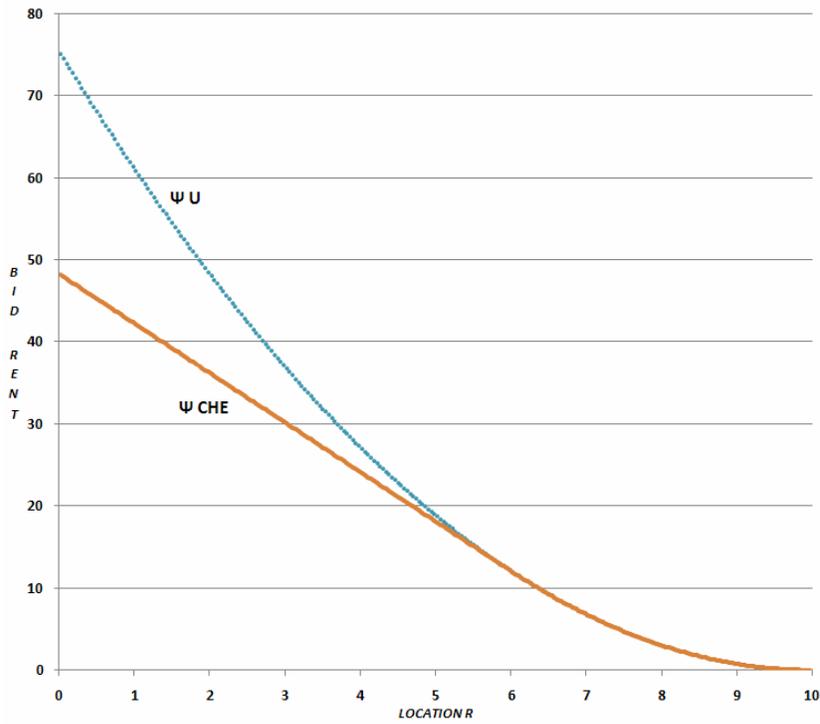

FIGURE 1b   Bid rent functions.

As previously observed, for a given utility the HE constraint reduces both the lot size and the bid rent inside the binding zone, and increases the consumption of the composite good. Outside the binding zone, we find the same solutions for the CHE and unconstrained models.

We are now ready to characterize the equilibriums.

PROPOSITION 4

In the applied case, the equilibrium is characterized as follows:

	$\alpha \leq \alpha_{cr}$	$\alpha \in [\alpha_{cr}, 1/2]$	$\alpha \geq 1/2$
$e^{2\tilde{u}}$	$\dfrac{\alpha^2 Y^2}{R_A^2}\left(\sqrt{a^2 N^2 + \left(\dfrac{1-\alpha}{\alpha}\right)^2 R_A^2} - aN\right)$	$\dfrac{Y^2\left(1 - 2\alpha + 2\alpha^2\right)}{2(aN + R_A)}$	$\dfrac{Y^2}{4(aN + R_A)}$
$\tilde{r}_f$	$\dfrac{Y}{a}\left(1 - \alpha\left(1 + \sqrt{\left(\dfrac{aN}{R_A}\right)^2 + \left(\dfrac{1-\alpha}{\alpha}\right)^2} - \dfrac{aN}{R_A}\right)\right)$	$\dfrac{Y}{a}\left(1 - \sqrt{\dfrac{R_A}{aN + R_A}\left(2 - 4\alpha + 4\alpha^2\right)}\right)$	$\dfrac{Y}{a}\left(1 - \sqrt{\dfrac{R_A}{aN + R_A}}\right)$

where $\alpha_{cr} = \left(1 + \sqrt{1 + \dfrac{2aN}{R_A}}\right)^{-1}$

Calculations are based on the distinction of 3 cases:

- $\alpha \geq 1/2$ yields the unconstrained model
- If $\alpha \in [\alpha_{cr}, 1/2]$, the edge of the city is beyond $r_{bind}(\alpha)$
- If $\alpha \leq \alpha_{cr}$, the HE constraint is active for the whole city

Comparative statics for the applied model

After computing the different equilibriums, we can proceed to a more precise analysis of the role of α.

Utility level   In the applied model, while an appropriate choice of α increases the households' utility compared to the unconstrained city, setting α to too low a value usually decreases it.

PROPERTY 2

For any given set of parameters $(N, Y, R_A > 0, a)$, the equilibrium utility $\tilde{u}(\alpha)$ of the CHE city strictly decreases on $[\alpha_{cr}, 1/2]$ with $\tilde{u}(1/2) = u_{eq}$. It is maximal for $\alpha_{max} < \alpha_{cr}$, with $\tilde{u}(\alpha_{max}) > u_{eq}$. Furthermore, $\tilde{u}(\alpha) \underset{\alpha \to 0}{\to} -\infty$

If $R_A = 0$, $\tilde{u}(\alpha)$ strictly decreases on $]0, 1/2]$ and therefore is maximal when α tends toward 0.

Demonstration (proof omitted) is carried out by using proposition 4.

Figure 2 depicts the variations of $e^{2\tilde{u}(\alpha)}$ for the reference model (corresponding to $N=10$, $Y=80$, $a=8$ and $R_A=20$); for these settings $\alpha_{cr}=0.25$.
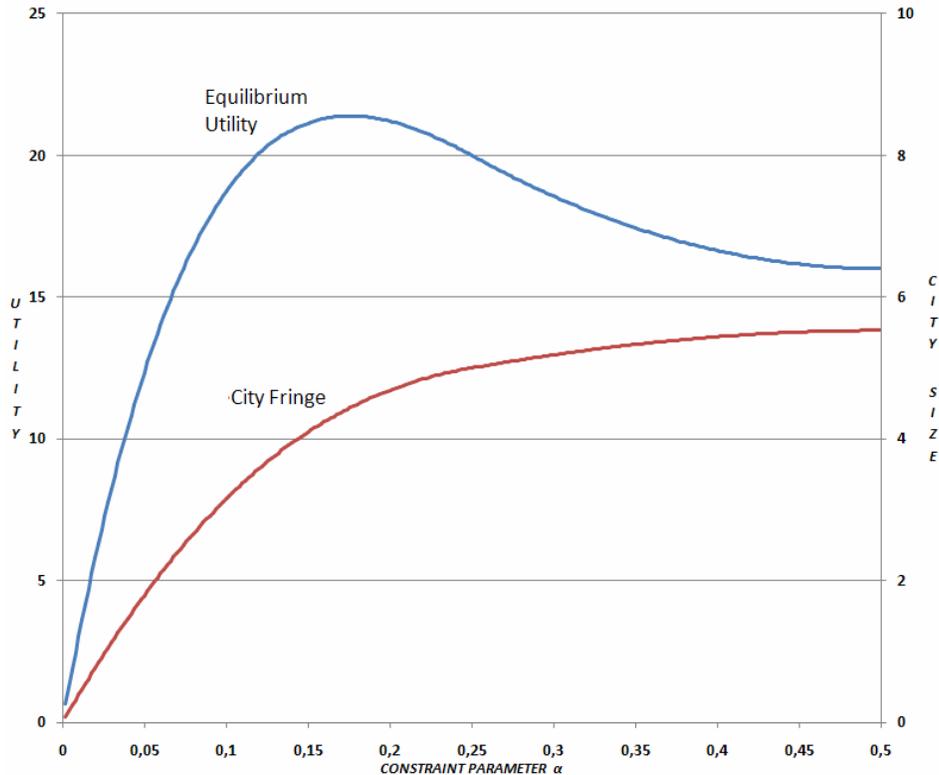

FIGURE 2 Utility level and city size of the CHE city.

We can check that $0.176=\alpha_{max}<\alpha_{cr}=0.25$, which corroborates property 2.

Property 2 confirms proposition 3: whenever the city fringe is beyond the binding zone (i.e. $\tilde{r}_f(\alpha) \geq r_{bind}(\alpha)$ which is equivalent to $\alpha \geq \alpha_{cr}$), the CHE city displays a higher utility level than the unconstrained city. On the other hand, if the city is entirely constrained, reducing α proves worthwhile at first but quickly utility dwindles.

In fact, when the outside competition for land (the agricultural sector) is mild, the constraints put on households' choices are more than compensated for by the drop in prices that results from less fierce competition for land within the binding zone. This increases the utility of all households. Conversely, if the competitiveness of the households is too weak relative to the agricultural sector, the reduction of the city size is exacerbated and leads to declining utility.

City Size and Density Unlike the utility level, tightening the housing budget constraint always reduces city size (see proposition 2) as shown on Figure 2. When α is decreased from 0.5 to 0, city size shrinks, and this phenomenon is accentuated when $\alpha<\alpha_{max}$, i.e. when the HE constraint becomes too significant relatively to the need to compete with the agricultural sector for land.

Reduction of the city size is achieved in different ways according to the value of α:

- When the utility level increases, owing to higher densities near the CBD that overweigh lower densities in the suburban area
- When the utility level decreases, density uniformly rises throughout the city

Figure 3 illustrates the equilibrium densities for the original city, and two CHE cities with $\alpha=0.3$ and $\alpha=0.15$:
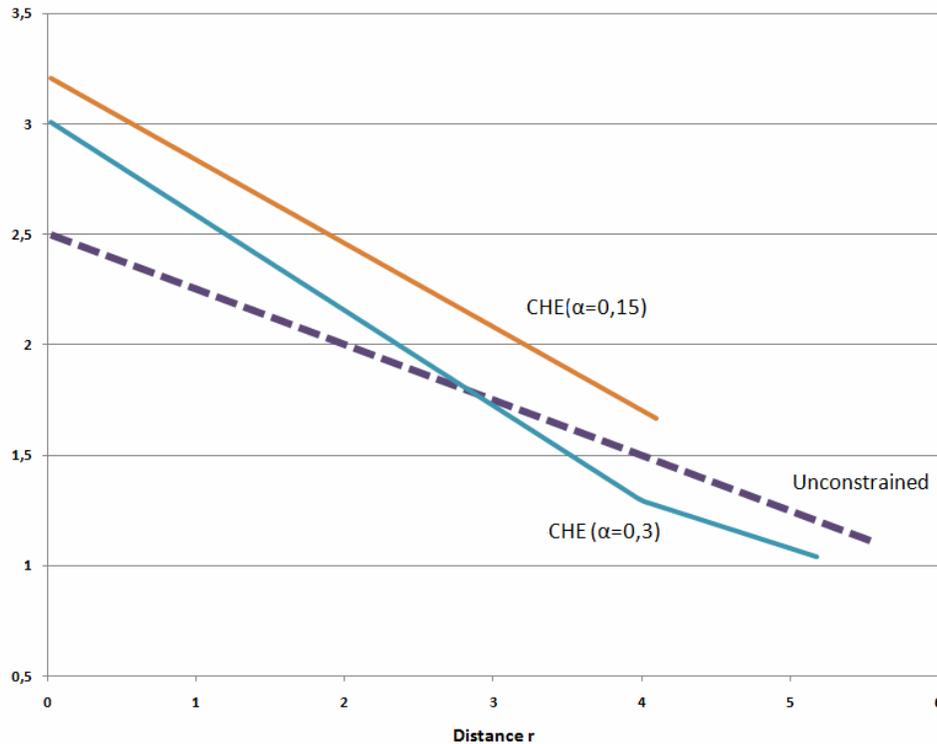

FIGURE 3 Influence of α on density in the reference model.

When α is chosen within $[\alpha_{max},1/2]$, we observe as predicted higher densities near the CBD, but lower densities in the suburbs. When α is chosen within $[0,\alpha_{max}]$, density rises throughout the whole town.

Average composition of household budgets   Since the HE policy was designed to cap housing expenses so as to ensure the solvency of the households, one key issue is the average composition of the household budget at equilibrium land use (proof omitted):

PROPERTY 3

For any given set (N,Y,R_A,a), the average expenditures for both housing and transportation are rising with α, inducing a declining consumption of the z good

Figure 4 exemplifies Property 3 for the reference model. For high values of α (between 0.4 and 0.5), decreasing α only slightly reduces the housing and transportation budget shares, because a limited number of households is affected by the constraint. If α further decreases (approximately until $\alpha_{max}=0.176$), housing expenses decrease more sharply while transport costs are moderately

affected. On this interval, decreasing α has a more significant depressing effect on prices than on lot sizes. Below α_{max}, the constraint weighs more on the households' choices of lot size, resulting in smaller cities and lower transportation and housing expenditures.
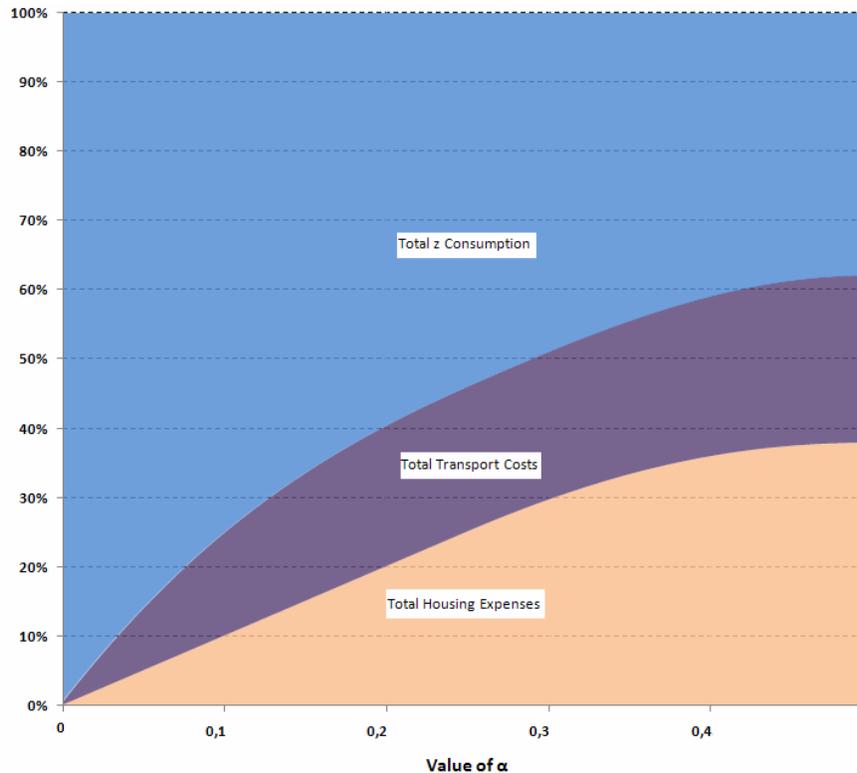

FIGURE 4 Influence of α on the average composition of the household's budget.

Concluding remarks for the CHE model   In sum, capping housing expenditures has the twofold effect of distorting households' residential choices (regarding lot size) and reducing equilibrium prices of the housing market. At first, the latter effect overweighs the former, leading to an increase in the utility level while the global structure of the city (size, use of transportation) remains relatively unchanged. Nevertheless decreasing α further eventually drastically reduces lot sizes, resulting in a drop in both utility level and city size.

Of course, the increase in utility generated by ad hoc values of α has a cost: the total housing expenses distributed to landlords (more precisely, the adequate notion would be the total differential land rent presented in (9) but to maintain the simplicity of the argument I refer to total housing expenses). By enforcing reduced prices, the CHE policy proceeds to a form of redistribution from the landlords to the households similar to the public ownership case described in Fujita (9), where rents are redistributed to the households. This redistribution is at the origin of the higher utility than in the unconstrained city with absentee landlords.

Given the analysis of the Herbert-Stevens model (9), we know that utility of the closed-city model is maximized in the case of public ownership. No other configuration of the city, and in particular the CHE city, can outperform this one in utility grounds. Yet, the CHE policy is

widely enforced and accepted, while such is not the case for the public ownership of land. Thus it is an interesting policy that can improve the solvency of the households and increase their utility at the same time, though being detrimental to landlords.

CONSTRAINING THE SHARE OF BOTH HOUSING AND TRANSPORTATION

Let us now turn our attention to an alternative policy, consisting in capping the total share of housing and transportation expenditures. As previously, I examine the impacts of such a policy on the equilibrium city, in particular the influence of the constraint parameter μ.

Considering the similarities between the CH+T and CHE policies, I first present the main results, omitting the proofs, and then focus on the application to the linear city.

The Constrained Housing+Transportation (CH+T) model

Overview of the CH+T model

The CH+T model is a monocentric model amended with the following additional constraint:

$$R(r)s + T(r) \leq \mu Y \qquad\qquad (E9)$$

The sum of housing and transportation expenditures is capped to within a fraction μ of the household's income *Y*. The case $\mu \geq 1$ is consequently tantamount to the classic unconstrained model.

Enforcement of such a policy yields the same effects as the CHE policy:

- constraining lot size choices of the households (actually it sets a *de facto* a minimal density)
- lowering prices

Yet this time it can be shown that the constraint concerns above all the households in the suburban area (starting from the edge of the city). The tighter it becomes, the more households it affects until covering the whole city.

Equilibrium features in the general case

The CH+T land use equilibrium exists and is unique. The only specific property of the equilibrium in the general case is that city size increases with μ, which is the result of the minimal density enforcement. The HT constraint induces the two same economic forces that influence the equilibrium utility level:

- By obliging the households to make sub-optimal choices, the latter achieve a lower utility level

- But capping HT expenses generates a "discount" on housing prices, which is beneficial to the households

Nevertheless, unlike the HE policy, there is no obvious case where we can predict the outcome. The same goes for housing expenses.

Application to a linear city

In order to compare the CHE and CH+T policies, let us come back to the application where: $U(z,s) = 1/2 \log z + 1/2 \log s$, $T(r) = ar$ and $L(r) = 1$.

Derivation of the equilibrium city

Determining the binding zone   The HT constraint is strictly binding when:

$$r > r_{bind}(\mu) = \frac{(2\mu - 1)Y}{a} \tag{E10}$$

Hence the following cases:

- If $\mu < 1/2$ the HT constraint is always binding.
- If $\mu \geq 1/2$, households located beyond $r_{bind}(\mu)$ are bound by the HT constraint.

Characterization of the equilibrium   Resolution of the bid-max program brings about the following system of equations:

$$r \leq r_{bind}(\mu) \begin{cases} \hat{z}(r,u) = (Y - ar)/2 \\ \hat{s}(r,u) = e^{2u} / \hat{z}(r,u) \\ \hat{\Psi}(r,u) = e^{-2u}(Y - ar)^2 / 4 \end{cases} \qquad r \geq r_{bind}(\mu) \begin{cases} \hat{z}(r,u) = (1 - \mu)Y \\ \hat{s}(r,u) = e^{2u} / \hat{z}(r,u) \\ \hat{\Psi}(r,u) = e^{-2u}(1 - \mu)Y(\mu Y - ar) \end{cases} \tag{E11}$$

Figure 5 illustrates (*E11*) for the settings of the reference model (N=10, Y=80, a=8, R_A=20). Moreover I choose μ=0.70 and u=16 (corresponding to the equilibrium utility of the CH+T reference model for the selected value of μ), which yields r_{max}=7 and r_{bind}=4.
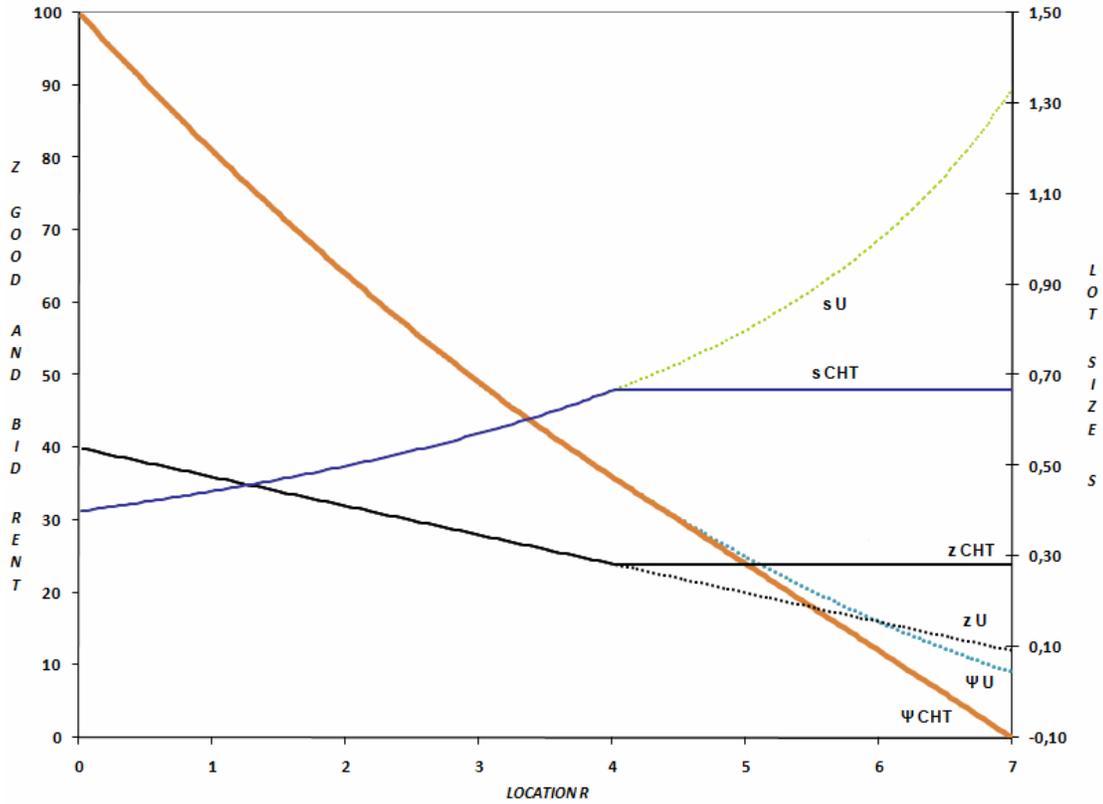
FIGURE 5 **Bid Rent, Surfaces and z Good Consumptions in the Unconstrained (U) and CH+T models.**

As previously stated, for $r \leq r_{bind}$ the HT constraint is ineffective, leading to the same solution that in the original case. For $r \geq r_{bind}$, the constraint becomes active, leading to constant values for lot size and z good consumption.

From (*E11*), we can derive the equilibrium utility and city size of the CH+T city:

PROPOSITION 5

In the applied case, the equilibrium is characterized as follows:

	$\mu \leq 1/2$	$\mu \in \left[1/2, \mu_{cr}\right]$	$\mu \geq \mu_{cr}$
$e^{2\hat{u}}$	$\dfrac{\mu(1-\mu)Y^2}{aN + R_A}$	$\dfrac{Y^2}{4(aN + R_A)}$	$\dfrac{Y^2}{4(aN + R_A)}$
$\hat{r}_f$	$\dfrac{\mu Y}{a}\dfrac{aN}{aN + R_A}$	$\dfrac{Y}{a}\left\{\mu - \dfrac{1}{4(1-\mu)}\dfrac{R_A}{(aN + R_A)}\right\}$	$\dfrac{Y}{a}\left(1 - \sqrt{\dfrac{R_A}{aN + R_A}}\right)$

$$\text{where } \mu_{cr} = 1 - \frac{1}{2}\sqrt{\frac{R_A}{aN + R_A}}$$

Calculations are based on three distinct cases:

- $\mu \geq \mu_{cr}$ yields the unconstrained model.
- If $\mu \in [1/2, \mu_{cr}]$, $r_{bind}(\mu) \geq 0$, thus households living in the central area of the city are unconstrained.
- If $\mu \leq 1/2$, $r_{bind}(\mu) \leq 0$. The HT constraint is active for the whole city.

Comparative statics for the applied model

Utility level   Starting from $\mu=1$, while decreasing μ has no impact at first on the utility level of the households (compared to the unconstrained city), for $\mu \leq 1/2$ it decreases the utility level.

PROPERTY 4

For any given set of parameters *(N,Y,R_A,a)*, the equilibrium utility $\hat{u}(\mu)$ of the CH+T city strictly increases with μ on $[0,1/2]$ and is constant for $\mu \geq 1/2$.

Considering Proposition 5, Property 4 is straightforward. Yet, this property proves enlightening for it states that, for $\mu \in [1/2, \mu_{cr}]$, the capped lot sizes perfectly compensate for the "discount" on housing prices given to the households. If $\mu \leq 1/2$, the constraint becomes too strong, inducing a drop in the utility level.

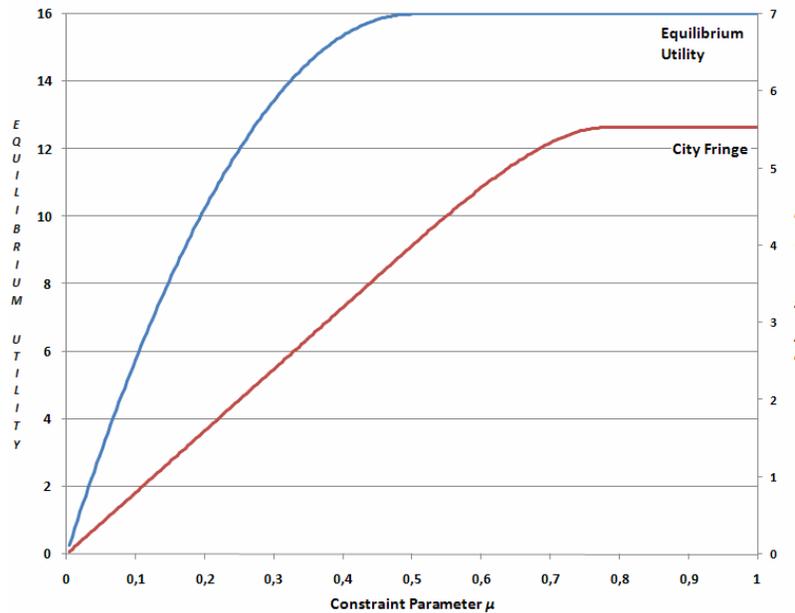Figure 6 depicts the variations of $e^{2\hat{u}(\mu)}$ for the reference model, where $\mu_{cr}=0.776$:

FIGURE 6 Utility level and city size of the CH+T city.

City Size and Density As previously stated for any given set of parameters (N,Y,R_A,a), the city size increases with μ, which is illustrated for the reference model in Figure 6. On $[1/2,\mu_{cr}]$ the city size is fairly well approximated by a linear function, which demonstrates the efficiency of this policy in reducing the city size (relatively to the CHE policy).

Similarly to the CHE policy, the CH+T policy alters the spatial distribution of density, but this time it sets a minimum density level that affects either the most remote part of the city ($\mu \in [1/2, \mu_{cr}]$), or the whole city ($\mu \leq 1/2$). This phenomenon is illustrated in Figure 7:
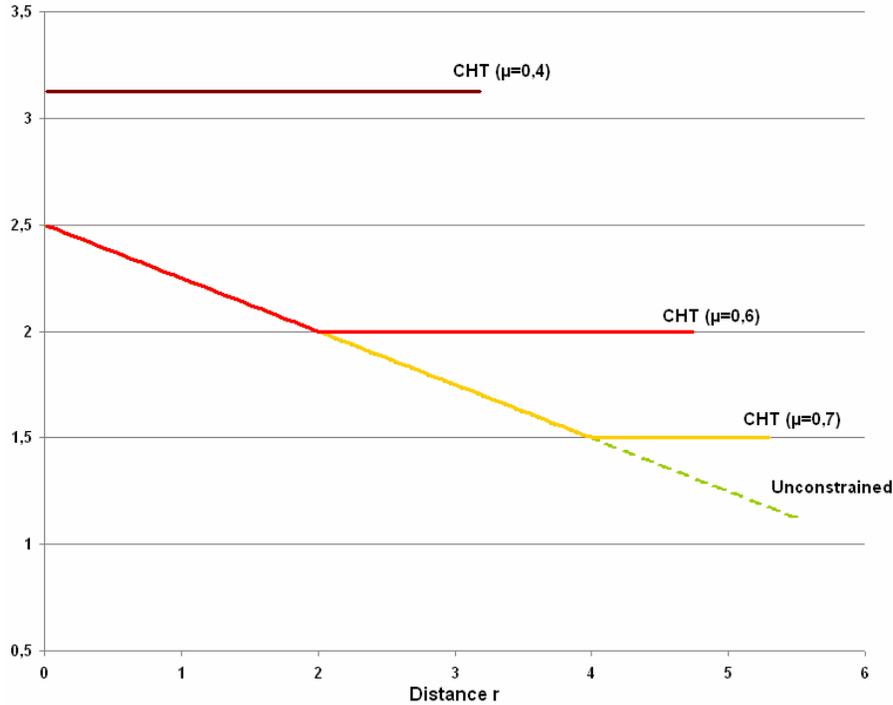

FIGURE 7 Influence of μ on equilibrium density in the reference model.

Average composition of the household budgets Similarly to the HE policy, the HT policy brings about lower housing and transportation expenditures for the households, as illustrated in Figure 8 (proof of following property omitted):

PROPERTY 5

For any given set (N,Y,R_A,a), the average expenditures for both housing and transportation increase with μ, while the average consumption of the composite good decreases with μ.

When the HT policy becomes active (starting from μ_{cr}), increasing the constraint results in decreasing transport costs and housing expenses. Unlike the HE policy, the two items decrease simultaneously in similar proportions, which results from capping housing and transportation expenses instead of only housing expenditures. When μ falls below 1/2, the decrease steepens.
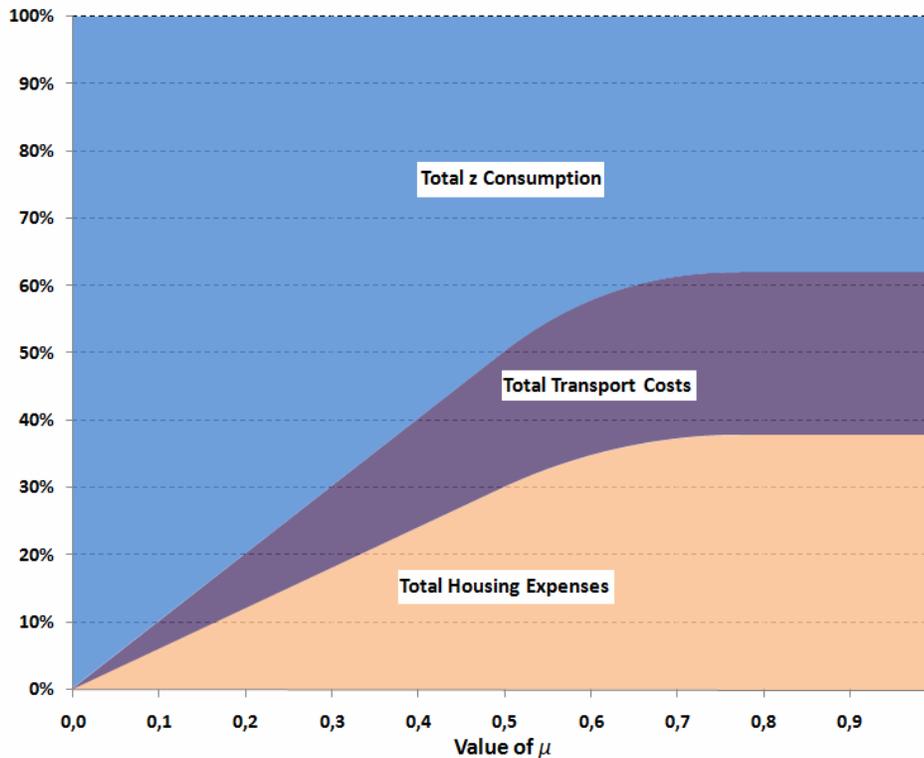
FIGURE 8 Influence of μ on the average composition of the household's budget.

CONCLUSIONS

Let us compare the main results concerning a linear city implementing either the CHE or the CH+T policy. I shall present the equilibrium utility level and city size for the reference applied model with a target solvency level, defined as the fraction of income remaining after paying the housing and transportation costs (Figure 9).

In both models, increasing the solvency of households is done by tightening the corresponding constraint, until reaching the maximal solvency level of 100% for a value of the constraint parameter equal to zero. Since a constraint parameter of one yields the unconstrained model in both cases, each pair of curves starts at the same point.

Figure 9 reveals that the CHE policy provides a greater utility for any target level of solvency, but at the cost of a greater city size. Regarding land use, while both policies induce shrinkage of the city, the CHE policy steepens the density curve when the utility rises, while the CH+T policy always flattens the density curve. Moreover, the CH+T policy is more efficient in reducing the city size, and consequently transportation costs and energy consumption.
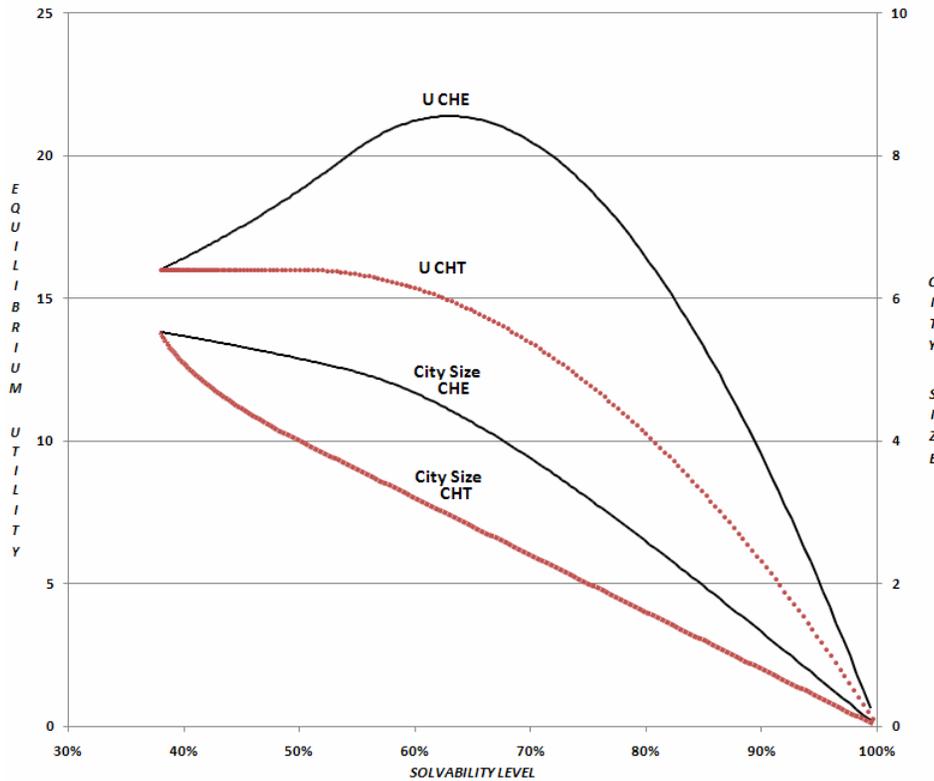
FIGURE 9 Comparison of utility levels and city sizes according to solvency level

Consequently, the linear model suggests that while the CHE policy is beneficial to households on utility grounds, and does improve their solvency while simultaneously reducing city size and transport expenses, the CH+T policy makes a better tool to struggle against urban sprawl and transportation costs. Because the model includes neither externalities such as pollution or congestion, nor the scarcity of energy, the CH+T policy might prove a better choice than the CHE policy depending on the objectives of the local authorities, and this despite utility considerations. In all cases, both policies can be used to secure a target level of solvency for the households.

While the model developed in the present paper was helpful in understanding the CHE and CH+T policies, several improvements are planned to assess the effects of these policies in more realistic settings:

- Considering the case of a disk-shaped city, which will complicate the calculations.
- Calibrating the utility functions and the parameters against existing metropolitan areas.
- Considering the policy impacts in terms of car ownership decision and modal choice, especially for the CH+T policy.

ACKNOWLEDGMENTS

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